

• 19. *The Principle of the Common Cause*

We shall now study some further applications of macrostatistics which lead once more to the distinction of cause and effect and thus to the definition of a time direction. The results will be formulated in a specific principle governing macroscopic arrangements. It will be shown, however, that this principle does not represent a new assumption, but is derivable from the second law of thermodynamics, if this law is supplemented by the hypothesis of the branch structure.

Suppose that lightning starts a brush fire, and that a strong wind blows and spreads the fire, which is thus turned into a major disaster. The coincidence of fire and wind has here a common effect, the burning over of a wide area. But when we ask why this coincidence occurred, we do not refer to the common effect, but look for a common cause. The thunderstorm that produced the lightning also produced the wind, and the improbable coincidence is thus explained.

The schema of this reasoning illustrates the rule that the improbable should be explained in terms of causes, not in terms of effects. Let us postpone an investigation concerning the relationship between this inference and the hypothesis of the branch structure, and let us first study the inference in its own right. The logical schema which governs it may be called the *principle of the common cause*. It can be stated in the form: *If an improbable coincidence has occurred, there must exist a common cause.*

In our daily life we often employ inferences of this kind. Suppose both lamps in a room go out suddenly. We regard it as improbable that by chance both bulbs burned out at the same time, and look for a burned-out fuse or some other interruption of the common power supply. The improbable coincidence is thus explained as the product of a common cause. The common effect, the fact that the room becomes completely dark, cannot account for the coincidence. Or suppose several actors in a stage play fall ill, showing symptoms of food poisoning. We assume that the poisoned food stems from the same source—for instance, that it was contained in a common meal—and thus look for an explanation of the coincidence in terms of a common cause. There is also a common effect of the simultaneous illness of the actors: the show must be called off, since replacements for so many actors are not available. But this common effect does not explain the coincidence.

Chance coincidences, of course, are not impossible: the bulbs may burn out simultaneously, the actors become ill simultaneously for different reasons. The existence of a common cause is therefore in

such cases not absolutely certain, but only probable. This probability is greatly increased if coincidences occur repeatedly. Suppose two geysers which are not far apart spout irregularly, but throw up their columns of water always at the same time. The existence of a subterranean connection of the two geysers with a common reservoir of hot water is then practically certain. The fact that measuring instruments such as barometers always show the same indication if they are not too far apart, is a consequence of the existence of a common cause—here, the air pressure. Further illustrations would be easy to find.

It will be advisable to treat the principle of the common cause as a statistical problem. For this purpose we assume that A and B have been observed to occur frequently; thus it is possible to speak of probabilities $P(A)$, $P(B)$, and $P(A.B)$ with reference to a certain time scale. For instance, we may count the eruptions of the geysers within a scale of days, or hours, and thus arrive at the probability that the geysers will spout on a given day, or at a given hour. We shall at the same time generalize the problem by extending it to statistical relationships between cause and effect, that is, to situations in which the cause produces the effect only with a certain probability. The special case that this probability is practically equal to 1, as in the illustrations given, is then included in the general treatment.

The statistical relationships which two simultaneous events have, on the one hand, to a common cause, and, on the other hand, to a common effect, can be presented by the schema of figure 23. We have here a double-fork arrangement in which A and B represent the two events the simultaneous occurrence of which is improbable; C is their common cause; E , their common effect. We said that C , and not E , explains the simultaneous occurrence of A and B . Let us therefore omit the upper part of the diagram and study the single fork of figure 24, in which merely the common cause C is indicated.

That the simultaneous happening of A and B is more frequent than can be expected for chance coincidences, can be written in the form

$$P(A.B) > P(A) \cdot P(B) . \quad (1)$$

When we apply to the left side the general multiplication theorem of probabilities,

$$P(A.B) = P(A) \cdot P(A, B) = P(B) \cdot P(B, A) , \quad (2)$$

we derive from (1) the two relations

$$P(A, B) > P(B) , \quad (3)$$

$$P(B, A) > P(A) . \quad (4)$$

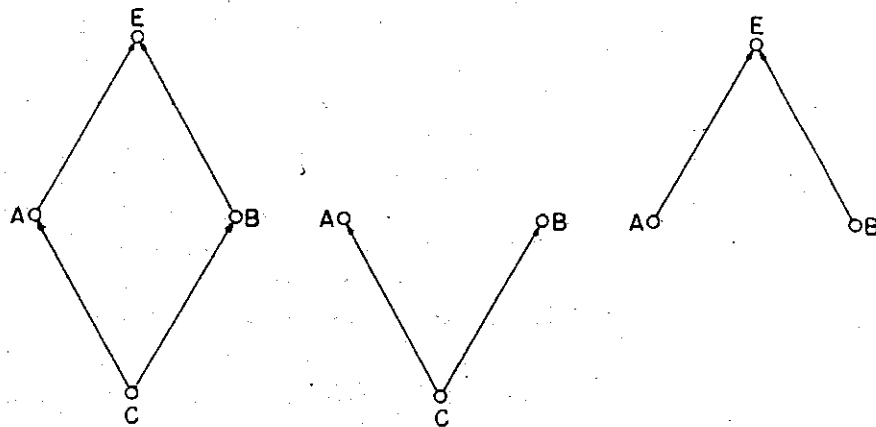


Fig. 23 (left). Double fork, constituted by a common cause and a common effect.

Fig. 24 (center). Fork open toward the future, constituted by a common cause.

Fig. 25 (right). Fork open toward the past, constituted by a common effect.

Each of these relations, vice versa, can be used to derive (1). Therefore (3), or (4), is equivalent to (1). In these derivations, it is always assumed that none of these probabilities vanishes.

In order to explain the coincidence of A and B , which has a probability exceeding that of a chance coincidence, we assume that there exists a common cause C . If there is more than one possible kind of common cause, C may represent the disjunction of these causes. We will now introduce the assumption that the fork ACB satisfies the following relations:

$$P(C, A.B) = P(C, A) \cdot P(C, B) , \quad (5)$$

$$P(\bar{C}, A.B) = P(\bar{C}, A) \cdot P(\bar{C}, B) , \quad (6)$$

$$P(C, A) > P(\bar{C}, A) , \quad (7)$$

$$P(C, B) > P(\bar{C}, B) . \quad (8)$$

It will be shown presently that relation (1) is derivable from these relations. For this reason, we shall say that relations (5)-(8) define a *conjunctive fork*, that is, a fork which makes the conjunction of the two events A and B more frequent than it would be for independent events. When we say that the common cause C *explains* the frequent coincidence, we refer not only to this derivability of relation (1), but also to the fact that relative to the cause C the events A and B are mutually independent: a *statistical dependence* is here derived from an

independence. The common cause is the connecting link which transforms an independence into a dependence. The conjunctive fork is therefore the statistical model of the relationship formulated in the principle of the common cause.